DECONFINING BY WINDING

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A model for the quantum effective description of the vacuum structure of thermalized SU(3) Yang-Mills theory is proposed. The model is based on Abelian projection leading to a Ginzburg-Landau theory for the magnetic sector. The possibility of topologically non-trivial, effective monopole fields in the deconfining phase is explored. These fields are assumed to be Bogomol'nyi-Prasad-Sommerfield saturated solutions along the compact, euclidean time dimension. Accordingly, a gauge invariant interaction for the monopole fields is constructed. Motivated by the corresponding lattice results the vacuum dynamics is assumed to be dominated by the monopole fields. A reasonable value for the critical temperature is obtained, and the partial persistence of non-perturbative features in the deconfining phase of SU(3) Yang-Mills theory, as it is measured on the lattice, follows naturally.

1 Abelian projection, monopole trajectories, and field theoretic description

Quantum Chromodynamics (QCD) and SU(N) Yang-Mills (YM) theory are asympotically free (regime of large momenta)[a], and they exhibit a growth of the coupling toward the infrared, perturbatively. The latter is a *necessary* condition for a successful explanation of the fact that the non-collective propagation of color charges over large distances has not been observed experimentally. Due to lattice results this is commonly believed to be caused by the pecularities of the gauge boson interactions. Therefore, we restrict ourselves to the discussion of pure gluodynamics, that is, SU(N) YM theory. A promising attempt was made by Mandelstam and 't Hooft in the late seventies and early eighties [1]. Roughly, they proposed to view the QCD and YM vacuum as a dual superconductor which forces the chromo-electric flux between a pair of largely separated test charges into a tube characterized by a constant tension σ. The corresponding potential grows linearly with separation, and hence color charges are confined. How can one obtain a condensate of magnetic charges responsible for this? The idea is that due to a presumably educated gauge fixing the low-energy degrees of freedom of SU(N) Yang-Mills

[a] For QCD this statement is only true if the number of active quark flavors is does not exceed a critical value set by the number of colors.

become transparent. Imposing a gauge condition invariant under the maximal Abelian subgroup $\mathrm{U}(1)^{\mathrm{N}-1}$, the emergence of chromo-magnetic monopoles can be observed. For example, demanding that the homogeneously transforming, hermitian field strength component $F_{23}(x)$ be diagonal after gauge fixing is no constraint for a gauge transformation$\in\mathrm{U}(1)^{\mathrm{N}-1}$. Moreover, for a given configuration A_μ there may be points in space where the eigenvalues of F_{23} are degenerate. At such a location the entire gauge group is unconstrained by the gauge condition. With respect to *Abelian* components of the gauge field A_μ one encounters radially directed magnetic flux with the magnetic field becoming singular at the singularity of the gauge fixing procedure and along a half-line starting there. A magnetic monopole together with its Dirac string is recovered. However, the statement that at the instant x^0 there sits a magnetic monopole at point $\vec{x}$ is a *highly gauge variant* and arbitrary one. Instead of diagonalizing $F_{23}(x)$ under gauge transformations we could have chosen to diagonalize $F_{12}(x)$. Generically, this would have caused the gauge singularities to be distributed in a different way. This is a serious problem, and one only can hope that the dynamics itself proliferates the gauge *invariant* existence of monopoles at low resolution. Another unanswered question arises in view of the monopole interpretation using Abelian components of the gauge fields A_μ: What justifies the projection onto Abelian components? The only hint that this procedure describes the low-resolution physics properly comes from the lattice where the string tension indeed was found to be saturated by the projected dynamics and, even better, by the monopole dynamics alone [5,6,7]. Analytical insight into this miracle may be obtained if the vacuum at low resolution can be shown to be governed by the dynamics of an adjoint Higgs model with the gauge field being essentially pure gauge [8].

For now let us be pragmatic. Point-like monopoles in space correspond to line-like trajectories in spacetime. It can be shown that a summation over these trajectories for monopole charge $|Q|=1$ in the partition function leads to a scalar field theory for the monopole sector [2]. However, working with the projected gauge field alone the theory necessarily contains non-local terms which one would like to avoid [2]. The introduction of a twin set of gauge fields can cure this [2,3]. Projecting QCD, one obtains a theory of electrically and magnetically charged matter fields and sets of (N-1) electric and magnetic abelian gauge fields $\vec{A}_\mu$ and $\vec{B}_\mu$, respectively. A *phenomenological* self-interaction of the magnetic monopole fields is assumed to yield a spontaneously generated condensate breaking the $\mathrm{U}(1)^{N-1}$ symmetry [4]. As a result, the set of gauge fields $\vec{B}_\mu$, interacting with the magnetic charges [b], becomes massive, and a

[b] This sector is a dual Ginzburg-Landau theory.

constant string-tension σ emerges.

2 The case of finite temperature

It has been shown on the lattice (for example [12]) that a thermalized SU(3) Yang-Mills theory undergoes a deconfinement phase transition at critical temperatures $T_c^{lat} \sim 200 - 300$ MeV. As explained above the starting point for an analytical investigation of this phenomenon is the dual Ginzburg-Landau theory given by the following Lagrangian [4]

$$\mathcal{L}_{DGL} = -\frac{1}{4}(\partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu)^2 + \sum_{k=1}^{3} \left\{ \left| (i\partial_\mu - g\vec{\varepsilon}_k \cdot \vec{B}_\mu)\phi_k \right|^2 - V_k(\phi_k, \bar{\phi}_k) \right\} \ . \tag{1}$$

In contrast to the electric, Abelian gauge group the magnetic $\mathrm{U}(1)^2$ (with gauge field $\vec{B}_\mu = (B^3_\mu, B^8_\mu)$) is believed to be spontaneously broken by nonvanishing VEVs of the monopole fields ϕ_k. In Eq. (1) $g\vec{\varepsilon}_i$ denote the effective magnetic charges with $\vec{\varepsilon}_1 = (1,0)$, $\vec{\varepsilon}_2 = (-1/2, -\sqrt{3}/2)$, $\vec{\varepsilon}_3 = (-1/2, \sqrt{3}/2)$, and the fields ϕ_k satisfy the constraint $\sum_{k=1}^{3} \arg \phi_k = 0$ [4].

The potential $V \equiv \sum_{k=1}^{3} V_k(\phi_k, \bar{\phi}_k)$ is introduced *phenomenologically* to account for the self-interaction of the monopole fields ϕ_k. In the framework of the dual Ginzburg-Landau theory the deconfinement phase transition at finite temperature has been first discussed in Ref. [10]. In Ref. [11] the critical temperature T_c of the deconfinement phase transition was determined as the point where a thermal one-loop effective potential, calculated in euclidean spacetime with compactified time dimension of size $\beta \equiv 1/T$, starts possessing an absolute minimum with vanishing monopole VEV's. For the sake of renormalizability and the desired feature of spontaneous breaking of the magnetic gauge symmetry the potential V for the monopole fields was chosen to be Higgs-like: $\lambda \sum_{k=1}^{3} \left(\bar{\phi}_k \phi_k - v^2\right)^2$.
However, since the Abelian description of gluodynamics can only be valid up to a certain resolution Λ_b[c] one may wonder whether a renormalizable monopole interaction is imperative. Introducing a pair of external, oppositely charged, static, electric color-charges and integrating out the electric and (massive) magnetic gauge bosons and the monopole fluctuations about their VEV's in quadratic approximation a critical tempertaure $T_c \sim 500$ MeV was obtained from the corresponding effective potential. This is too high. The result is not surprising since the effective potential was calculated by abusing perturbation theory ($\lambda \sim 25$ from a fit to the string tension σ!).

[c] After all perturbative QCD works well for large momenta.

Our approach is therefore different. We view the monopole potential to be a quantum effective potential already. With the lattice-motivated assumption that the vacuum dynamics is *dominated by the monopole fields* it is sufficient to look for solutions to the classical equations of motion of the monopole sector. The potential is then constructed such that
1) it is gauge invariant under the magnetic $\mathrm{U}(1)^2$,
2) it admits topologically non-trivial, BPS saturated solutions along the compact, euclidean time dimension [d], and
3) it allows for *one* topologically trivial, non-vanishing VEV for the monopole fields at zero temperature.
Thereby, the requirement of BPS saturation derives from the fact that we are interested in a vacuum description. The corresponding fields must then saturate the lowest bound for the euclidean action. From **1)** and **2)** it follows that the "square root" $V_k^{1/2}$, defined as $V_k(\bar\phi_k\phi_k) \equiv V_k^{1/2}(\bar\phi_k)V_k^{1/2}(\phi_k)$, must have a single pole at $\phi_k = 0$ [13,14,16]. **3)** enforces an analytical part in order to obtain finite VEV's at zero temperature (stabilization). Furthermore, this analytical part can only be a power $\sim \phi^N$ since a genuine polynomial would introduce non-degenerate vacua. From **1)** it finally follows that

$$V_k(\phi_k,\bar\phi_k) == \lim_{N\to\infty}\left\{\frac{\Lambda^6}{\bar\phi_k\phi_k} + \kappa^2\Lambda^{-2(N-2)}(\bar\phi_k\phi_k)^N - 2\,\kappa\Lambda^{5-N}\frac{1}{\bar\phi_k\phi_k}\mathrm{Re}\,\phi_k^{N+1}\right\}\,. \tag{2}$$

Thereby, Λ is a mass-parameter, and κ is some dimensionless coupling constant. Considering the rhs of Eq. (2) at finite N, the potential explicitely breaks the magnetic $\mathrm{U}(1)^2$ gauge symmetry, $\vec B_\mu \to \vec B_\mu + 1/g\,\partial_\mu\vec\theta(x)$, $\phi_k \to \mathrm{e}^{-i\vec\varepsilon_k\cdot\vec\theta(x)}\phi_k$, down to Z^2_{N+1} due to the term $\mathrm{Re}\,\phi_k^{N+1}$ [16]. Only in the limit of large N is the gauge symmetry restored. For the purpose of qualitative illustration Fig. 1 shows the potential V_k. Note that at $T = 0$ this effective potential V ideed does not allow for fluctuations of the fields ϕ_k due to its infinite curvature at $\phi_k = \Lambda$. The BPS equations, corresponding to the potential of Eq. (2), read

$$\partial_\tau\phi_k = \bar V^k_{1/2}(\bar\phi_k)\ ,\quad \partial_\tau\bar\phi_k = V^k_{1/2}(\phi_k)\ . \tag{3}$$

The rhs are only fixed up to phase factors $\mathrm{e}^{i\delta}$, $\mathrm{e}^{-i\delta}$, respectively. From Eq. (2)

[d] Recall, that in the euclidean description finite temperature is implemented by compactifying the time dimension.

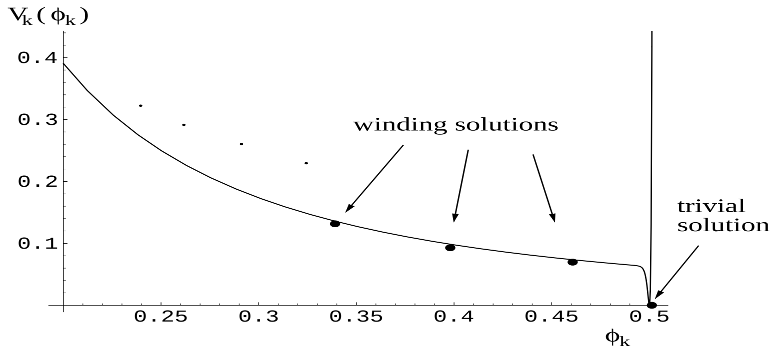

Figure 1. The monopole potential V_k as a function of real ϕ_k for $\kappa = 1$, $\Lambda = 0.5$, and $N = 500$. Indicated are the potential energy densities of topologically trivial and non-trivial solutions to the BPS equations.

we have for *periodic* solutions

$$V_{1/2}^k(\phi_k) = \begin{cases} \pm i\frac{\Lambda^3}{\phi_k} \ , & (|\phi_k| < \Lambda) \\ 0 \ , & (|\phi_k| = \Lambda) \\ \infty \ , & (|\phi_k| > \Lambda) \end{cases} \ . \tag{4}$$

3 BPS saturated winding solutions

For the case $|\phi_k| < \Lambda$ periodic solutions ($\phi_k(0) = \phi_k(\beta)$) to Eqs. (3) subject to Eq. (4) have been discussed in Refs. [13,14] within the framework of supersymmetric theories. For a compact, euclidean time dimension of length $\beta \equiv 1/T$ the set of topologically distinct solutions is

$$\phi_k^{n_k}(\tau) = \sqrt{\frac{\Lambda^3\beta}{2|n_k|\pi}}\,\mathrm{e}^{2n_k\pi i\frac{\tau}{\beta}} \ , \quad (n_k \in \mathbf{Z}) \ . \tag{5}$$

Thereby, the sign of n_k corresponds to the choice of phase in the BPS equations. Due to the phase constraint the sets of solutions $(\phi_1^{n_1}, \phi_2^{n_2}, \phi_3^{n_3})$ can be labeled by the integers n_1 and m, which are both odd *or* even. The winding numbers n_2, n_3 are then given as $-n_1/2 \mp m/2$, respectively. The gauge function $\vec{\theta}_{n_1,m}(\tau)$, transforming to the unitary gauge $\mathrm{Re}\,\phi_k(\tau) > 0$, $\mathrm{Im}\,\phi_k(\tau) = 0$, reads

$$\vec{\theta}_{n_1,m}(\tau) = \frac{2\pi}{\beta}\,\tau \begin{pmatrix} n_1 \\ \frac{m}{\sqrt{3}} \end{pmatrix} \ . \tag{6}$$

Note that this non-periodic function leaves the periodicity of the gauge field $\vec{B}_\mu$ intact.

4 Deconfinement phase transition

Eqs. (3) admit the solution $\phi_k^0 \equiv \Lambda$ with winding number and energy density zero, and the winding solutions of the previous section. We identify the set $(\phi_1^0, \phi_2^0, \phi_3^0)$ with the confining vacuum at low temperatures. In this regime the spectrum consists of glueballs with masses that are much larger than the prevailing temperatures [9]. The thermal equilibrium between the vacuum "medium" and its excitations is essentially realized at pressure zero, which indeed is observed on the lattice [18]. Above the deconfinement phase transition the ground state "medium" readily emits and absorbs (almost) free gluons under the influence of the heat bath like a black body emits and absorbs photons. We identify the genuine winding set of lowest potential energy density[e], represented by $(n_1, m) = (2, 0)$, with the ground state just above the deconfinement transition. In this picture the confining vacuum is a perfect thermal insulator up to the transition, where its structure drastically changes by the absorption of an amount of latent heat per volume [18] equal to the gap $\Delta\varepsilon^{(2,0)}$ between the potential V of the zero winding and the lowest genuine winding set. We estimate the critical temperature T_c of this transition by assuming the deconfining vacuum to behave like an incompressible, static fluid with traceless energy-momentum-tensor [17] (no scale anomaly) which is in thermal equilibrium with an ideal gas of gluons. In this case we have the equation of state $\varepsilon = 3p$ for pressure p and energy density ε for both the vacuum (vac) and the gluon gas (gg). The equilibrium condition of equal pressures, $p_{vac} = p_{gg}$, then takes the following form [17]

$$\Delta\varepsilon^{(2,0)} = \frac{8}{15}\pi^2 T_c^4 \ , \quad \text{where} \quad \Delta\varepsilon^{(2,0)} = 8\pi\Lambda^3 T_c \ . \tag{7}$$

Using Eqs. (7) and the condition $p_{vac} = p_{gg}$, we obtain $T_c = \left(\frac{15}{\pi}\right)^{\frac{1}{3}} \Lambda \sim 1.68\,\Lambda$. Adopting the value $\Lambda = 0.126$ GeV for the monopole condensate at $T = 0$ from Refs. [9,11], this yields $T_c = 0.212$ GeV which is compatible with the lattice results of $T_c = 0.2 - 0.3$ GeV.

[e] By "genuine" we mean that each of the solutions ϕ_k is winding. Note, however, that there are also semi-winding sets, for example $(n_1, m) = (1, 1)$, which contain two winding fields and one field of winding number zero. Since none of the monopole fields ϕ_k should be singled out it is natural to assume that either all ϕ_k are winding or none at all.

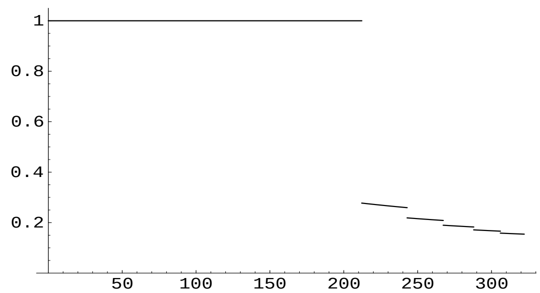

Figure 2. The average monopole field $\phi_{av}(T)$ in units of $\phi_{av}(T=0)$. For further explanation see text.

5 The deconfining phase

Transforming the winding solution of Eq. (5) to the unitary gauge by means of Eq. (6), we obtain the following expression for the individual condensate, corresponding to winding n_k: $\phi_k^{n_k} = \frac{\Lambda^{3/2}}{\sqrt{2\pi|n_k|T_l}} = \frac{\Lambda}{\sqrt{(30\pi^2 l)^{1/3}|n_k|}}$[f]. For the configuration $(n_1, m) = (2, 0)$ we obtain $\phi_{av}^{(2,0)} \sim 0.28\,\Lambda$. In Fig. 2 the evolution of the average monopole condensate is depicted up to $T \sim 0.34$ GeV. In unitary gauge the monopole fields are real, and the spontaneous breakdown of the $\mathrm{U}(1)^2$ magnetic gauge symmetry then transparently generates a mass m_B for the field $\vec{B}_\mu$ which scales linearly with the average monopole condensate, $m_B^{(n_1,m)} = \sqrt{3}\, g\, \phi_{av}^{(n_1,m)}$. In Refs. [9,11] the value $m_B \sim 0.5$ GeV was determined at $T = 0$, yielding a magnetic gauge coupling of $g = 2.3$.
How does the string tension σ evolve? An analytical expression for σ in terms of g, m_B, and m_ϕ was derived in Ref. [9] by calculating the potential between two heavy and static quarks of opposite color charge, separated by a distance R. Thereby, an effective Lagrangian for the dynamics of the electric $U(1)^2$ gauge field $\vec{A}_\mu$ is obtained by integrating out the massive, magnetic vector field $\vec{B}_\mu$ and by assuming that the monopole fields do not fluctuate about their VEVs. Applying convenient gauge fixing terms and introducing external, static color currents, which couple to $\vec{A}_0$, one obtains an effective action. From this action one extracts the potential energy $V_{\bar{q}q}(R)$ and reads off as $\sigma =$

[f] $l \equiv \sum_k |n_k|$.

$2\pi m_B^2/(3g^2)\ln\left[1+m_\phi^2/m_B^2\right]$. Since *static* test charges were assumed finite temperature does not necessitate any new considerations. The appearance of the monopole mass $m_\phi > m_B$ under the logarithm in the expression for σ is due to its role as an ultraviolet cutoff for the integration over transverse momenta in the expression for the linear part of the potential $V_{\bar{q}q}(R)$. This is justified by the observation that $m_B \sim 0$ inside the flux tube of radius $\rho \sim m_\phi^{-1}$ in a type II dual superconductor (see Ref. [11] and Refs. therein). Since we work with effective fields ϕ_k we cannot asign a mass to the fluctuations of the monopole fields about the background of a classical ground state. We may, however, assume that ρ^{-1} scales with temperature in the same way as m_B does. Since the dependence of σ on the cutoff is logarithmic the result is not dramatically sensitive to the exact T dependence of ρ^{-1}, which we will make explicit by distinguishing the following cases:
(a) $\rho^{-1}(0) = \rho^{-1}(T) = 1.26$ GeV [9] $\Rightarrow$

$$\frac{\sigma(T_c)}{\sigma(0)} \sim (0.28)^2 \frac{\ln(1+(1.26)^2/(0.5\times 0.28)^2)}{\ln(1+(1.26)^2/(0.5)^2)} \sim 0.19\ . \tag{8}$$

(b) $\rho^{-1}(T) \propto m_B(T) \Rightarrow$

$$\frac{\sigma(T_c)}{\sigma(0)} \sim (0.28)^2 \sim 0.08\ . \tag{9}$$

Hence, the discrepency amounts to a factor 2 for these two extreme cases.

6 Summary and discussion

We considered a thermalized dual Ginzburg-Landau theory modelling hot SU(3) Yang-Mills theory. From the postulate that in contrast to the confining phase the ground state of the deconfining phase is characterized by topologically nontrivial, BPS saturated solutions to the classical equations of motion of the monopole sector and the uniqueness of the vacuum at $T=0$ we deviced the corresponding, gauge invariant interaction. As a consequence, the average monopole field undergoes a drastic decrease to about 1/4 of its zero temperature value across the phase boundary. Scaling linearly with the monopole condensate, the same applies to the mass of the magnetic vector fields generated by the spontaneous breaking of the magnetic $U(1)^2$ gauge symmetry. Working with the zero temperature value for the monopole condensate of Refs. [9,11], we obtain a reasonable critical temperature $T_c \sim 212$ MeV in contrast to the effective potential calculation of Ref. [11], where $T_c \sim 500$ MeV, and the more realistic value of $T_c \sim 200$ MeV was obtained by introducing an ad hoc T-dependence of the dimensionless coupling constant λ.

Depending on the assumption about the T-dependence of the flux tube radius ρ the string tension σ decreases to about 1/12..1/5 of its value at $T = 0$ across the phase boundary which is compatible with lattice measurements [18]. To determine T_c we assumed a *free* gluon gas although close to the critical T_c the lattice indicates a pressure in the deconfining phase which is sizably smaller than the Stefan-Boltzmann asymptotics. However, since the leading term in an expansion of p in (mass scale)/T is quartic in T we expect T_c to be robust against changes in the subleading terms[g]. The presence of non-perturbative effects is described by non-vanishing values of the monopole condensate ϕ_{av} and the string tension σ, and the model predicts a slow decrease of these quantities. On the lattice and in effective potential calculations the deconfinement phase transition has been determined to be of first order [11,18,19]. In contrast, SU(2) Yang-Mills theory exhibits a second order phase transition [19]. The model of the present work can be easily adapted to the dual Ginzburg-Landau theory describing maximal abelian gauge fixed SU(2) Yang-Mills theory. The qualitative results would then be the same. It is clear that we cannot resolve this difference since by definition the order parameter $|\phi_{av}|$ is always discontinuous. However, the model gives a reason for the residual, non-trivial structure of monopole condensation in SU(2) and SU(3) Yang-Mills theories at high temperatures, it predicts a reasonable value for the critical temperature T_c, and it incorporates the low-temperature limit in the form of a constant solution to the BPS equations. The existence of a non-trivial structure of monopole condensation above the phase transition has been pointed out in Ref. [20] a long time ago based on lattice investigations.

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[g] If there were no subleading terms T_c would be given by a cubic root.

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